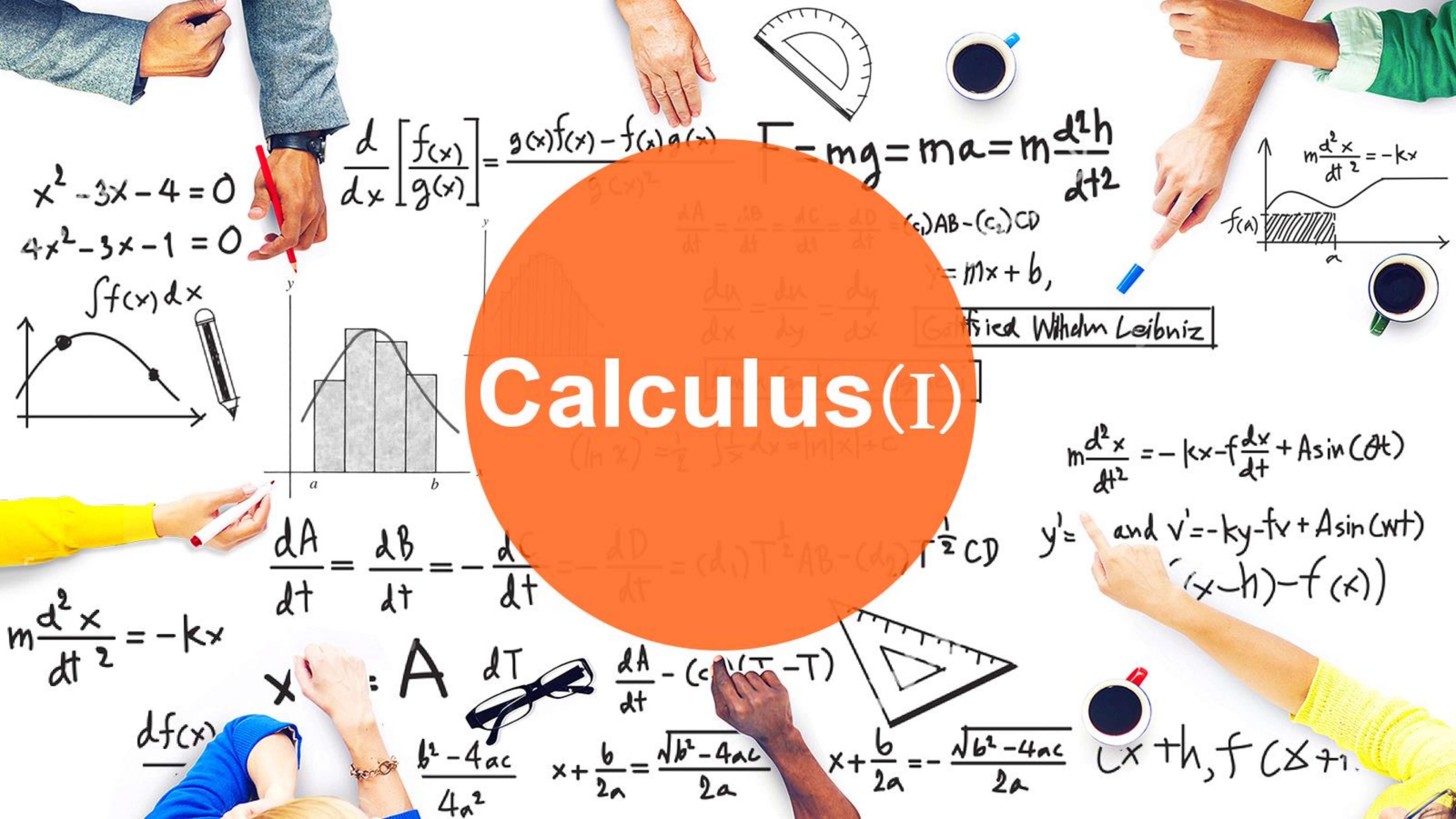

$x^2 - 3x - 4 = 0$
$4x^2 - 3x - 1 = 0$
$\int f(x)\,dx$
$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] =$
$= mg = ma = m\frac{d^2h}{dt^2}$
$m\frac{d^2x}{dt^2} = -kx$
$= mx + b,$
Gottfried Wilhelm Leibniz
$m\frac{d^2x}{dt^2} = -kx - f\frac{dx}{dt} + A\sin(\theta t)$
and $v' = -ky - fv + A\sin(wt)$
$\frac{dA}{dt} = \frac{dB}{dt} = -$
$m\frac{d^2x}{dt^2} = -kx$
$\frac{b^2 - 4ac}{4a^2}$
$x + \frac{b}{2a} = \frac{\sqrt{b^2 - 4ac}}{2a}$
$x + \frac{b}{2a} = -\frac{\sqrt{b^2 - 4ac}}{2a}$

Calculus(Ⅰ)

Limits

Lecturer: Xue Deng

infinitesimal

Comparison of infinitesimal

infinitesimal equivalence

Comparison of infinitesimal

Def: If the limit of $\alpha(x)$ is 0, then the expression $\alpha(x)$ is called infinitesimal.

If, when $x \to 0$, x, x^2, $\sin x$, $x^2 \sin\frac{1}{x}$ infinitesimal.

All kinds of limit

$\lim\limits_{x\to 0}\frac{x^2}{3x} = 0$, $x^2 \to 0$ is faster than $3x \to 0$;

$\lim\limits_{x\to 0}\frac{\sin x}{x} = 1$, $\sin x \to 0$ and $x \to 0$ are almost the same;

$\lim\limits_{x\to 0}\frac{x^2 \sin\frac{1}{x}}{x^2} = \lim\limits_{x\to 0}\sin\frac{1}{x}$ does not exist.

Definition

(1) $\lim \frac{\beta}{\alpha} = 0,\quad \beta$ is the higher order infinitesimal than α

$\beta = o(\alpha);$

(2) $\lim \frac{\beta}{\alpha} = \infty,\quad \beta$ is the lower order infinitesimal than α

(3) $\lim \frac{\beta}{\alpha} = C(C \neq 0),\quad \beta$ is the same order infinitesimal as α

$C = 1,\ \beta$ and α are infinitesimal equivalence $\alpha \sim \beta.$

(4) $\lim \frac{\beta}{\alpha^k} = C \quad (C \neq 0, k > 0),\quad \beta$ is the k-order infinitesimal of α

Examples

(1) $n \to \infty$, $\frac{1}{n^2}$ is the higher order infinitesimal than $\frac{1}{n}$ $\frac{1}{n^2} = o\left(\frac{1}{n}\right)$;

(2) $x \to \infty$, $\frac{1}{x}$ is the same order infinitesimal as $\frac{100}{x}$

(3) $x \to 0$, $\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$,

$1 - \cos x$ is the 2-order infinitesimal of x

Infinitesimal equivalence $\quad x \to 0$

$\sin x \sim x,\quad \arcsin x \sim x,\quad \tan x \sim x,$

$\arctan x \sim x,\quad \ln(1+x) \sim x,\quad e^x - 1 \sim x,$

$\sqrt{1+x} - 1 \sim \frac{1}{2}x,\quad \sqrt[n]{1+x} - 1 \sim \frac{1}{n}x,$

$1 - \cos x \sim \frac{1}{2}x^2.$

Examples

Eg 1： Prove: $x \to 0, 4x\tan^3 x$ is the 4-order infinitesimal of x.

$$\lim_{x\to 0}\frac{4x\tan^3 x}{x^4} = 4\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^3 = 4,$$

$\therefore x \to 0, 4x\tan^3 x$ is the 4-order infinitesimal of x.

Examples

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}$$

Eg 2: $x\to 0$, find the k-order of $\tan x-\sin x$ about x?

$$\because \lim_{x\to 0}\frac{\tan x-\sin x}{x^3} = \lim_{x\to 0}\left(\frac{\tan x}{x}\cdot\frac{1-\cos x}{x^2}\right) = \frac{1}{2},$$

$$= C\ (C\neq 0)$$

$\therefore$ $\tan x-\sin x$ is the 3-order infinitesimal of x.

Th: infinitesimal equivalence substitution theorem

Let $\alpha \sim \alpha'$, $\beta \sim \beta'$ and $\lim \frac{\beta'}{\alpha'} = A($ or $\infty)$,

Then $\lim \frac{\beta}{\alpha} = \lim \frac{\beta'}{\alpha'} = A($ or $\infty)$.

$$\lim \frac{\beta}{\alpha} = \lim\left(\frac{\beta}{\beta'} \cdot \frac{\beta'}{\alpha'} \cdot \frac{\alpha'}{\alpha}\right)$$

$$= \lim \frac{\beta}{\beta'} \cdot \lim \frac{\beta'}{\alpha'} \cdot \lim \frac{\alpha'}{\alpha}$$

$$= \lim \frac{\beta'}{\alpha'} = A(\text{or } \infty).$$

Examples

Eg 1: $\lim\limits_{x\to 0}\dfrac{\tan 2x}{\sin 5x}.$

It is convenient to find the limit of $\dfrac{\mathbf{0}}{\mathbf{0}}$

$$x \to 0,\quad \tan 2x \sim 2x,\quad \sin 5x \sim 5x,$$

$$\lim_{x\to 0}\frac{\tan 2x}{\sin 5x} = \lim_{x\to 0}\frac{2x}{5x} = \frac{2}{5}.$$

Examples

Eg 2: $\lim\limits_{x \to 0} \dfrac{\tan^2 2x}{1-\cos x}$.

$$x \to 0,\ 1-\cos x \sim \frac{1}{2}x^2,\ \tan 2x \sim 2x.$$

$$\lim_{x \to 0} \frac{\tan^2 2x}{1-\cos x} = \lim_{x \to 0} \frac{(2x)^2}{\frac{1}{2}x^2} = \mathbf{8.}$$

only as some factor.

Examples

Eg 3: $$\lim_{x\to 0}\frac{\tan x-\sin x}{\sin^3 2x}$$

Wrong $x \to 0,\quad \tan x \sim x,\quad \sin x \sim x,$

$$L = \lim_{x\to 0}\frac{x-x}{(2x)^3} = 0.$$

$x \to 0,\quad \sin 2x \sim 2x,$

$$\tan x-\sin x = \tan x(1-\cos x) \sim \frac{1}{2}x^3,$$

$$\mathrm{L} = \lim_{x\to 0}\frac{\frac{1}{2}x^3}{(2x)^3} = \mathbf{\frac{1}{16}}.$$

Question

$$\lim_{x\to 0}\frac{\tan 5x-\cos x+1}{\sin 3x}$$

A: $\mathbf{Ans}=\dfrac{5}{3}$.

$\tan x \sim x,\quad \sin x \sim x,$

$1-\cos x \sim \dfrac{1}{2}x^2$

Question

$\ln(1+x) \sim x, \quad \sin x \sim x, \quad \tan x \sim x,$

$$\lim_{x\to 0}\frac{\ln\sqrt{1+x}+2\sin x}{\tan x}$$

$$\lim_{x\to 0}\frac{\ln\sqrt{1+x}+2\sin x}{\tan x}$$

$$=\lim_{x\to 0}\frac{\ln\sqrt{1+x}}{\tan x}+\lim_{x\to 0}\frac{2\sin x}{\tan x}$$

$$=\frac{1}{2}\lim_{x\to 0}\frac{\ln(1+x)}{\tan x}+2\lim_{x\to 0}\frac{\sin x}{\tan x}=\frac{5}{2}$$

Limits